

Anomaly-induced effective action for gravity and inflation

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Abstract. In the very early Universe the matter may be described by the free radiation, that is by the set of massless fields with negligible interactions between them. Then the dominating quantum effect is the trace anomaly which comes from the renormalization of the conformal invariant part of the vacuum action. The anomaly-induced effective action can be found with accuracy to an arbitrary conformal functional which vanishes for the special case of the conformally flat metric. This gives the solid basis for the study of the conformally-flat cosmological solutions, first of which was discovered by Mamaev and Mostepanenko and by Starobinski in 1980. Treating the anomaly-induced action as quantum correction to the Einstein-Hilbert term we explore the possibility to have inflationary solutions, investigate their dependence on the initial data and discuss the restrictions in considering the density perturbations. The shape of inflationary solutions strongly depends on the underlying gauge model of the elementary particles physics. Two special cases are considered: Minimal Standard Model and the matter sector of $N = 8$, $D = 4$ supergravity. It turns out that inflation is almost inevitable consequence of the great difference between Planck mass and the mass of the heaviest massive particle.

1 Introduction.

The standard cosmological model [1] was constructed soon after the creation of General Relativity and it describes the present stage of the Universe pretty well (see, for example, [2]). At the same time this model fails to explain the early period of the evolution, and hence some new theoretical idea is required. It is commonly accepted that problems of initial singularity, horizon, flatness and monopoles may be solved if one supposes that the early Universe passed through the period of a very fast (more than the speed of light) inflation when its size increased in more than 60 orders. Moreover, the origin of cosmological perturbations has an elegant explanation within such scenario. On the other hand, the source of inflation still remains unclear. In the first works devoted to the exponential expansion of the Universe the origin of it was considered in the quantum effects of vacuum [3, 4] (see also [5]), but the real explosion of interest to it started after the paper [6] where inflation was induced as a

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consequence of the Spontaneous Symmetry Breaking in the Standard Model of electroweak interactions. The further development has shown that the last cosmological scenario was not perfect and many other models, theories and approaches were considered. Finally it was realized that all the requirements to the inflationary model can be satisfied only if one introduces some new scalar field called inflaton, which has a potential of a very special form (see, for example, [7, 8, 9] for a comprehensive review).

Here we intend to go backward to the historic origin of inflation [3, 4] and derive it from the quantum effects of pure radiation on curved background. It will be shown that the inflationary solution doesn't require any fine-tuning or special choice of the parameters of the underlying conformal invariant matter fields but is absolutely natural and exists for a wide set of initial data. The exit from the inflationary phase may be also naturally explained by the nonstability of the exponential solution with respect to the density perturbations that can be relevant at the end of inflationary epoch. In this letter our consideration is restricted by the conformally flat metrics, but this does not mean that the similar results can not be achieved for the $k = +1$ or $k = -1$ cases.

The paper is organized as follows. In the next section we consider the model for the very Early Universe filled by the free radiation. This radiation is composed by the set of free, massless, conformally invariant matter fields on an arbitrary curved background. We review the derivation of the quantum correction to the classical action of gravity [10, 11]. This correction results from the trace anomaly [12] and it contains an arbitrary conformal invariant functional. However, for the conformally flat background metric this functional is irrelevant and hence in the framework of the above model the solution of [10, 11] is exact. In section 3 the particular inflationary solution is presented and its nonstability is established. Section 4 contains the results of numerical investigation and the physical analysis of the inflation. In the last section we draw some conclusions and outline the prospects for further studies.

2 Free fields and quantum corrections

The action of the ideal liquid filling the Universe can be written as

$$S_M = \int d^4x \sqrt{-g} \left\{ -\frac{1}{2} (p + \rho) u^\mu u^\nu g_{\mu\nu} + \frac{3}{2} p + \frac{1}{2} \rho \right\}. \quad (1)$$

It is easy to see that for the conformally flat cosmological metric the variation of this action with respect to the conformal factor of the metric

$$\frac{\delta S_M}{\delta a} = (3p - \rho) a^3 V$$

vanishes for $p = \frac{\rho}{3}$, and therefore $T^\mu_\mu = 0$, so the radiation should be described by the conformal invariant field actions.

In the framework of the asymptotically free gauge theories the interactions between matter fields are weakened in the high energy limit. Therefore we start from the action of free massless conformal invariant fields: N_0 scalars (spin-0), $N_{1/2}$ spinors (Dirac, spin-1/2) and N_1 abelian vectors (spin-1). All N 's indicate a number of fields (not multiplets) in

curved space-time, taking conformal coupling with scalar curvature for the scalars.

$$S_{matter} = \int d^4x \sqrt{-g} \left\{ \sum_{i=1}^{N_0} \frac{1}{2} (\nabla^\mu \varphi_i \nabla_\mu \varphi_i + \frac{1}{6} R \varphi_i^2) + i \sum_{j=1}^{N_{1/2}} \bar{\psi}_j \gamma^\mu \nabla_\mu \psi_j - \frac{1}{4} \sum_{k=1}^{N_1} F_{k,\mu\nu} F_k^{\mu\nu} \right\}. \quad (2)$$

For the free fields on curved background the only possible divergences are the one-loop vacuum ones. Therefore the renormalizability requires the action of vacuum to be introduced in the form (see [13] and [14] for the introduction)

$$S_{vacuum} = \int d^4x \sqrt{-g} \{ l_1 C^2 + l_2 E + l_3 \square R \} \quad (3)$$

where $l_{1,2,3}$ are some new (with respect to the flat space-time) parameters, C^2 is square of the Weyl tensor and E is the integrand of the Gauss-Bonnet topological (in $d = 4$) invariant. The vacuum action (3) includes only conformal invariant and surface terms. For the convenience of the reader we reproduce known formulas

$$R_{\mu\nu\alpha\beta}^2 = 2C^2 - E + \frac{1}{3} R^2, \quad R_{\mu\nu}^2 = \frac{1}{2} (C^2 - E) + \frac{1}{3} R^2$$

One has to notice that the introduction of conformal non-invariant terms like $\int \sqrt{-g} R$ or $\int \sqrt{-g} R^2$ is possible but not necessary for the renormalization of the free conformal invariant theories. The renormalization of vacuum leads to the conformal (trace) anomaly [12] which enables one to find the nonconformal part of the effective action of vacuum [10, 11]. For the sake of completeness and also to fix notations and signs we reproduce the main steps of this derivation.

The form of the one-loop divergences is well-known (see [13, 14]).

$$\bar{\Gamma}_{div}^{(1)} = - \frac{\mu^{D-4}}{(D-4)} \int d^Dx \sqrt{-g} \{ w C^2 + b E + c \square R \}. \quad (4)$$

Here D is the parameter of dimensional regularization and⁴

$$\begin{aligned} w &= \frac{1}{(4\pi)^2} \left(\frac{1}{120} N_0 + \frac{1}{20} N_{1/2} + \frac{1}{10} N_1 \right), \\ b &= - \frac{1}{(4\pi)^2} \left(\frac{1}{360} N_0 + \frac{11}{360} N_{1/2} + \frac{31}{180} N_1 \right), \\ c &= \frac{1}{(4\pi)^2} \left(\frac{1}{180} N_0 + \frac{1}{30} N_{1/2} - \frac{1}{10} N_1 \right). \end{aligned} \quad (5)$$

The contribution of the Weyl spinor is half of the Dirac one. We remark that the last two (topological and surface) terms in (4) are very important because they contribute to the trace anomaly and thus affect the dynamical equations for the effective action of gravity. The one-loop effective action has the form

$$\Gamma = S + \bar{\Gamma}, \quad \bar{\Gamma} = \bar{\Gamma}^{(1)} + \Delta S \quad (6)$$

⁴We use notation $R^\alpha_{\beta\gamma\delta} = \partial_\gamma \Gamma^\alpha_{\beta\delta} - \dots$.

where $\bar{\Gamma}^{(1)}$ is naive quantum correction to the classical action S and ΔS is a counterterm. The anomalous trace of the Energy-Momentum Tensor is [12]

$$T = \langle T^\mu_\mu \rangle = - \left[wC^2 + bE + c\Box R \right] \quad (7)$$

with the same coefficients a, b, c . The Eq. (7), in turn, gives rise to the equation for the finite part of the 1-loop correction to the effective action

$$\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta \bar{\Gamma}}{\delta g_{\mu\nu}} = wC^2 + bE + c\Box R \quad (8)$$

The solution of this equation is straightforward [10, 11] and one gets, in terms of $g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot \exp[2\sigma]$, the following quantum correction to the vacuum action ⁵.

$$\begin{aligned} \bar{\Gamma} = S_c[\bar{g}_{\mu\nu}] + \int d^4x \sqrt{-\bar{g}} \{ w\sigma \bar{C}^2 + b\sigma(\bar{E} - \frac{2}{3}\Box \bar{R}) + 2b\sigma \bar{\Delta}\sigma - \\ - \frac{1}{12} (c + \frac{2}{3}b) [\bar{R} - 6(\bar{\nabla}\sigma)^2 - 6(\Box\sigma)^2] \} . \end{aligned} \quad (9)$$

Here the fiducial metric $\bar{g}_{\mu\nu}$ has fixed determinant, Δ is conformal invariant self-adjoint operator

$$\Delta = \Box^2 + 2R^{\mu\nu} \nabla_\mu \nabla_\nu - \frac{2}{3} R \Box + \frac{1}{3} (\nabla^\mu R) \nabla_\mu$$

and $S_c[g_{\mu\nu}] = S_c[\bar{g}_{\mu\nu}]$ is some unknown conformal-invariant functional. In general, exact calculation of this functional is impossible. However, if we are interested in the conformally flat cosmological solutions, this functional is (fortunately) of no importance for us.

3 Particular inflationary solution

Our purpose is to implement the quantum correction (9) into the classical gravity-matter system. For the sake of simplicity here we consider only the conformally flat metrics, for which $S_c[\bar{g}_{\mu\nu}]$ is irrelevant. The solution (9) should be added to the classical action $S_{vacuum} + S_{matter}$. However, since we are interested in the conformally flat matter, it decouples from the conformally flat metric ⁶. Hence the only nontrivial contribution to the dynamical equation for σ comes from the vacuum part. The conformal and surface terms in the vacuum action (3) do not contribute to the dynamical equations for the conformal factor. On the other hand, in order to have correspondence with classical gravity one has to add the Einstein-Hilbert term to the vacuum action. Then we meet the total action of the form

$$S_t = -\frac{1}{\kappa^2} \int d^4x \sqrt{-g} R + \bar{\Gamma}[g] + \text{conformal invariant terms.} \quad (10)$$

⁵ Recently there were indications [15, 16] that the solution (9) does not assume the test based on the calculation of the three-point function, and hence another solution for $\bar{\Gamma}$ should be found. We emphasize that any solution for the effective action differs from (9) by the conformal invariant functional and this difference is not relevant for the cosmological solutions which will be explored here.

⁶ This fact can be seen, in particular, from the identity $T^\mu_\mu = 0$ which holds for the Energy-Momentum Tensor of the radiation.

and, denoting conformal time as η and taking conventional $\sigma = \ln a(\eta)$ we arrive at $\int d^4x \sqrt{-g} R = 6 \int d^4x a'^2$. Here the prime stands for the derivative with respect to η .

Let us remind that $\kappa^{-2} = M_{Pl}^2$. Taking variational derivative of the total action with respect to $a(\eta)$ one arrives at the equation of motion for $a(\eta)$:

$$\frac{a''''}{a} - 4 \frac{a' a'''}{a^2} - 3 \frac{a''^2}{a^2} + 2 \left(3 - \frac{2b}{c} \right) \frac{a'' a'^2}{a^3} + \frac{4b}{c} \frac{a'^4}{a^4} - \frac{2}{c} M_{Pl}^2 a'' a = 0. \quad (11)$$

It is convenient to rewrite (11) in terms of physical time t where $a(\eta)d\eta = dt$ so that $d/d\eta = a d/dt$ etc, and denote the derivatives with respect to t by points.

$$a^2 \ddot{\ddot{a}} + 3 a \dot{a} \ddot{a} - \left(5 + \frac{4b}{c} \right) \dot{a}^2 \ddot{a} + a \ddot{a}^2 - \frac{2M_{Pl}^2}{c} (a^2 \ddot{a} + a \dot{a}^2) = 0. \quad (12)$$

The last equation contains two dimensionless constants b, c which depend on the particle content of the matter fields and may be fixed by choosing particular gauge model. Also there is a dimensional constant M_{Pl} , which defines the scale. The Eq. (12) looks quite complicated and it is difficult to solve it exactly. Here we explore a particular exponential solution and in the next section perform the numerical study and physical analysis. Below we present all the numerical results for the Minimal Standard Model and for the compactified to $D = 4$ supergravity (M-theory). Indeed, since gravity is treated as purely classical background, in the last case one has to disregard all spin-2 and spin-3/2 degrees of freedom and keep only contributions of the spin-(0, 1/2, 1) sectors.

Let us look for the exponential solution of the form $a(t) = a_0(t) = A \cdot \exp(\lambda t)$ where A, λ are some constants. Substituting this expression into Eq. (12) one can easily see that it gives solution for an arbitrary A and $\lambda = \pm \frac{M_{Pl}}{\sqrt{-b}}$. Hence the crucial point is that b in (5) is negative for any particle content of the original theory (2). There are always two solutions with opposite signs for λ , and the positive one describes the exponentially expanding Universe. This exponential solution is exactly the one which has been obtained in [3, 4] by the use of the renormalized Energy-Momentum Tensor [17] (see also [5]). Some comment is in order. The equation for $a(t)$ which was used in [4] looks different from (12). This is because the in [4] the (0, 0)-component of the equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle$$

has been used while we use the trace of it. Indeed both equations contain the same information since the metric has only one degree of freedom. On the other hand, in our framework it is clear that the introduction of the conformal invariant matter fields (or the action for ideal liquid (1) with $p = \frac{\rho}{3}$) doesn't change the equation, so the above inflationary solution is valid for any (conformal) matter content⁷. An important consequence is that the density perturbations which do not violate the constraint $p = \frac{\rho}{3}$ can not destroy the exponential behavior of $a(t)$. On the other hand, any perturbations which do violate this constraint imply the appearance of the massive particles or cosmological constant. In both cases one can not consider these perturbations in the framework of the above model based on the conformal invariant fields (2) and conformal anomaly (7). If the initial theory contains some massive fields, than the trace $\langle T_{\mu}^{\mu} \rangle$ has non-anomalous (both classical and quantum)

⁷In this point our analysis differs from the one of [5].

contributions and one has to take them into account. Suppose the initial matter fields have masses of the order of 100 GeV that is typical for the Standard Model. Then this matter can be successfully considered as (2) at the energies a few orders below the Planck scale 10^{19} GeV and then the Universe is inflating until the typical energy becomes comparable with the masses. After that the density perturbations violating the constraint $p = \frac{\rho}{3}$ become relevant and they provide the successful exit from the inflationary phase and also basis for the formation of the galaxies [18]. Of course this scheme implies that below a Planck mass there is a sufficient gap on the mass scale, and the heaviest massive particle below M_{pl} is many orders lighter. If this condition is satisfied, one can regard all particles as massless, so that the constraint $p = \rho/3$ takes place. Then the most important questions concerning the exponential solution above are:

1. whether the inflationary solution is stable under the metric perturbations which are consistent with $p = \frac{\rho}{3}$?
2. whether the period of inflation is sufficient to provide the necessary rate of expansion?
3. whether the inflation occurs only for the very special initial data or it can be achieved in a more general situation? Indeed the above solution $a(t) = A \cdot \exp(\lambda t)$ implies that $a^{(n)}(0) = \lambda^n a(0)$. Hence this solution is only valid for some 1-dimensional line in the 4-dimensional space of the initial data.

In order to understand better the sense of the Planck mass in the above expressions, one can introduce the dimensionless time $\tau = M_{Pl} \cdot t$. The equation (12) remains almost the same, the only change is $M_{Pl} \rightarrow 1$. Then, in terms of the Planck units we have the following inflationary solution

$$a_0(t) = A e^{+\tau/\sqrt{-b}} \quad (13)$$

The Eq. (13) can be used to calculate the necessary duration of inflation. Suppose we want the Universe to expand in 10^n times, starting from some moment τ_0 . According to (13), the total rate of inflation is

$$\frac{a_0(\tau_0 + \Delta\tau)}{a_0(\tau_0)} = \exp \left\{ 4\pi \sqrt{\frac{360}{N_t}} \Delta\tau \right\}, \quad N_t = N_0 + 11 \cdot N_{1/2} + 62 \cdot N_1, \quad (14)$$

and thus we arrive at

$$\Delta\tau = \frac{\ln 10}{4\pi} \sqrt{\frac{N_t}{360}} \cdot n.$$

For the Minimal Standard Model $N_0 = 8$, $N_{\frac{1}{2}} = 48$, $N_1 = 12$ and we arrive at $\Delta\tau \approx 0.345 \cdot n$. Taking $n = 60$ one meets the necessary period of inflation $\Delta\tau \approx 20.7 t_{Pl}$ where t_{Pl} is the Planck time, which serves as a time unit for τ . Hence the necessary period of inflation is just one order higher than the Planck time. If one increases the particle content of the theory $\Delta\tau$ becomes longer. In particular, this happens when we use SU(5) GUT or the supersymmetric models. One can remind that the contributions of different fields to the coefficient b in (7) have the same signs. In general case the expected time of inflation can be evaluated as one-two orders longer than the Planck time, depending on the choice of the model.

As it was already mentioned above, it is important to investigate the metric perturbations around the solution (13). However this meets serious difficulty. While taking an arbitrary perturbations one can not avoid variations of the fiducial metric $\bar{g}_{\mu\nu}$. In this case one has to study this perturbations on the basis of the full action (10) and the conformal invariant terms may be as important as the anomalous correction which we deal with. In principle one can mimic the conformal action by the expressions described in [19] but we prefer to postpone this consideration for some future work. Here we consider only the perturbations of the conformal factor. It proves useful to rewrite the equation (12) for the $\sigma(\tau) = \ln a(\tau)$.

$$\ddot{\sigma} + 7\ddot{\sigma}\dot{\sigma} + 4\left(3 - \frac{b}{c}\right)\ddot{\sigma}\dot{\sigma}^2 + 4\dot{\sigma}^2 - 4\frac{b}{c}\dot{\sigma}^4 - \frac{2}{c}(2\dot{\sigma}^2 + \ddot{\sigma}) = 0 \quad (15)$$

and only then perform the perturbations of $\sigma(\tau) = \sigma_0(\tau) + x(\tau)$ near the inflationary solution $\sigma_0(\tau) = \ln A + \frac{1}{\sqrt{-b}}\tau$. For the sake of simplicity below we put $\ln A = 1$. The equation for the perturbations have the form

$$\ddot{x} + \frac{7}{\sqrt{-b}}\ddot{x} - \frac{2}{b}\left(6 - \frac{b}{c}\right)\ddot{x} - \frac{4}{c\sqrt{-b}}\dot{x} = 0 \quad (16)$$

(all derivatives with respect to τ) and can be easily explored. The stability of the inflationary solution under the perturbations of conformal factor strongly depends on the particle content $N_0, N_{1/2}, N_1$ of the initial action (2). We remark that the stability depends only on the ratio b/c . With the proportionally increasing number of fields $N_{0,1/2,1}$ the necessary period of inflation becomes longer but the (non)stability of solution (13) doesn't change.

Substituting $x = e^{r\tau}$ one meets three different nonzero roots $r_{1,2,3}$. We remark that the existence of the zero root manifests the arbitrariness of one of the coordinates in the 4-dimensional space of initial data (choice of $\ln A$). This can be also seen if one makes perturbations in (12) but in the last case a bit more sophisticated consideration is required. The values of $r_{1,2,3}$ depend on the particle content $N_{0,1/2,1}$. Taking the values typical for the Minimal Standard Model we arrive at

$$r_{1/2} \approx -9.99 \pm 24.71 i, \quad r_3 \approx -26.66. \quad (17)$$

Thus in the case of Minimal Standard Model all four roots have negative real parts. This means that the exponential inflation is stable under the small perturbations of the conformal factor of the metric. The possible sources of instability remain in the perturbations of the other degrees of freedom or in the density perturbations discussed above. It is interesting that the stability analysis performed (in a different way) in [4] gave different result. The source of the discrepancy is that in [4] the negative sign of c in (7) has been chosen. This sign, indeed, corresponds to the contributions of vector fields into (4), while spin-0 and spin-1/2 fields contribute with positive sign. Here we presented the results for the field content of the Standard Model, and will confirm our conclusion using numerical methods.

Let us consider, as a second example, the M-Theory, which is nowadays regarded as a possible candidate for the unified model of all interactions. In the framework of M-theory one possible realization is maximal $N = 1, D = 11$ supergravity. In this case its particle content is unique and the gauge group is $SO(8)$. Then the spin-0, 1/2, 1 sector of the compactified to $D = 4$ theory has 70 real scalars, 28 vectors and 56 Majorana spinors. Taking $N_{(0,1/2,1)} = (70, 28, 28)$ one meets the roots

$$r_{1/2} \approx -54.63 \pm 71.39 i, \quad r_3 \approx +72.63, \quad (18)$$

indicating to the instability of the exponential inflation. The question is whether this means instability of inflation at all? As we shall see in the next section, the answer is exactly opposite.

4 Numerical study and solutions with bounce

The fourth order equation (12) depends on two parameters b and c , the first being negative for any matter content, and c being positive or negative depending on the particle multiplet. Numerical integration of this equation depends on the value and sign of b and c and on the initial conditions. Among the many possible scenarios, we can identify three main ones:

1. Singular expanding Universe, with oscillations between accelerating and desaccelerating phases;
2. Inflationary expanding Universe;
3. Non singular Universes, presenting a contraction initial phase, followed by an expanding phase.

The exact inflationary solution (13) represents a particular choice of initial conditions. The perturbative study performed before indicates that this solution is stable for the particle content of the Standard Model (see Figures 1,2).

Since it is inconvenient to make a numerical study (and especially graphical presentation) of the Universe suffering from 60-orders inflation, we have explored the behavior of logarithmic variable $\sigma(\tau)$ rather than the $a(\tau)$ itself. It turns out that the inflation occurs for a very wide set of initial conditions, even when these conditions are taken far away from the exact exponential solution (13). In some cases, depending on the multiplet composition, the stable solutions correspond to some kind of "hiperinflation" and expansion perform much faster than exponentially.

A numerical study of the initial conditions around those representing the solution (13) reveals that this inflationary solution is "robust": there exist two main scenarios:

1. An eternal inflationary expansion;
2. A non singular Universe, presenting a bounce for the scale factor.

In the last case, the Universe exhibits an accelerating contraction followed by an accelerating expansion; just in some special cases there is a short non inflationary period of contraction (expansion). These numerical results confirm the analytical one, and extend the possible scenarios including Universe exhibiting bounce. Only these two scenarios were found for the Standard Model (Figures 1,2 give typical examples of the plots).

We remark that the cosmological scenario exhibiting bounces may be encounter in many non linear curvature models and in some special scalar-metric gravity theories. The main distinguishing feature of the scenarios obtained here is its almost universal inflationary behavior, while in the other theories there was just a short period of inflation near the bounce. The continuously expanding and bouncing solutions obtained here have the number of e -fold sufficient to solve the problems that can be cope with an inflationary phase.

As a second example we consider the multiplet of the M-Theory described in the previous section. In this case, if one performs a slight change of the initial conditions around those

typical for the exact inflationary solution, there can be an extremely fast expansion, with singularity just a fraction of (Planck) time unite after the initial moment. Some "typical" plots are presented at the Figures 3,4. One can call the behavior shown at Fig. 3 as "hyperinflation", while the one on Fig. 4 corresponds to some dramatic contraction of the Universe. Making the time inverse, however, we meet inflation also in this case. Solutions with the bounce are also possible.

5 Conclusions and discussions

We have considered the quantum effects of free radiation on the background of conformally flat metric and found that this leads, in a very natural way, to the inflation which must continue only a very short time to resolve the problems which inflation is supposed to resolve.

It is important to notice that the existence of exponential solution and the velocity of expansion depends only on $\int E$ -type topological counterterm and absolutely doesn't depend on the $\int \square R$ and $\int C^2$ -type counterterms (4). This may be important, since in the same manner as we have added $\int R$ to the vacuum action of the conformal theory, one could also add local (classical) $\int R^2$ -term with an arbitrary coefficient which can mixture with the same term in (9). That is why the $\int R^2$ -based inflation is not safe in general. But in the case of (13) everything depends only on the nonlocal term in the effective action (9) which does not suffer from this ambiguity. The only physical requirement which is important for inflation is a gap on the mass scale between Planck mass M_{pl} and heaviest massive particle M_{max} . In the course of inflation the typical energy scale decreases and, when it becomes comparable to M_{max} , the mass of the particles can be seen, the violations of $p = \frac{p}{3}$ constraint show up and the inflation quits due to the perturbations.

The remaining open question is indeed the stability of inflation with respect to arbitrary perturbations of the metric. As it was mentioned above, this question can not be addressed within the exact solution (9) because it is necessary to use some approximation (or imitation) for the unknown conformal invariant functional S_c . Similar situation takes place for the unconstrained density perturbations which can occur at small energies (or, in other terms, at later period of the evolution of the Universe). In this case one has to derive, in some reasonable approximation, the effective action of the massive fields and, as a best option, also take into account their interactions. In case of the weakly interacting conformal field one has to count the effect of back reaction of vacuum to the matter fields which produces a slow running from the conformal fixed point [20]. In this case one can explore the possibility to have inflationary solutions for the induced gravity. We are going to reconsider these problems elsewhere. The present letter contains a brief report on the possibility to have consistent and natural inflation from the vacuum effects and some review of its main features.

Acknowledgments. One of the authors (I.Sh) is grateful to J.A. Helayel-Neto for the discussion of the theory (18) and also acknowledges warm hospitality at the Departamento de Fisica of the Federal University of Juiz de Fora. A.M.P. is grateful to UFMG and FAPEMIG (MG-Brazil) for the scholarship and support. The work of J.C.F and I.L.Sh. has been supported by CNPq.

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FIGURE CAPTIONS

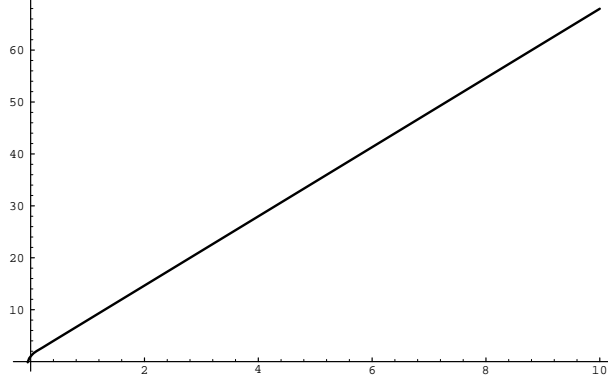


Figure 1: Typical inflationary behavior for the Standard Model case. Initial data: $\sigma(0) = 1$, $\dot{\sigma}(0) = 2m/\sqrt{-b}$, $\ddot{\sigma}(0) = 0$, $\ddot{\sigma}(0) = 0$.

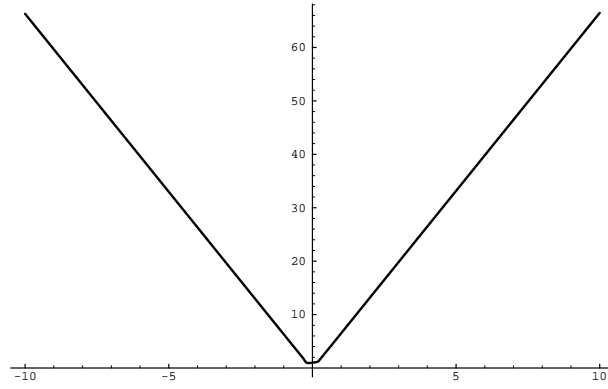


Figure 2: Another typical inflationary behavior for the Standard Model case. Initial data: $\sigma(0) = 1$, $\dot{\sigma}(0) = 0.1 \cdot m/\sqrt{-b}$, $\ddot{\sigma}(0) = 1$, $\ddot{\sigma}(0) = 0$.

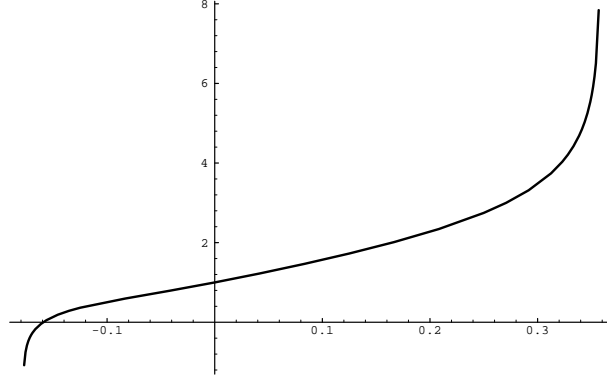


Figure 3: Typical inflationary (hyperinflationary) behavior for the M-theory case. Initial data: $\sigma(0) = 1$, $\dot{\sigma}(0) = m/\sqrt{-b}$, $\ddot{\sigma}(0) = 10$, $\ddot{\sigma}(0) = 0$.

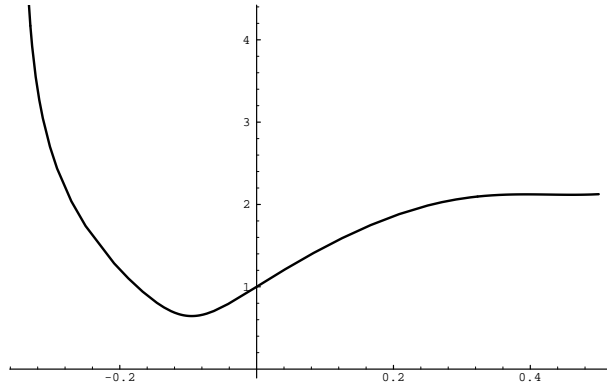


Figure 4: Non-inflationary behavior for the M-theory case. This kind of scenario with "hyperinflation" is possible only for large negative $\ddot{\sigma}(0)$. Initial data corresponding to this plot: $\sigma(0) = 1$, $\dot{\sigma}(0) = m/\sqrt{-b}$, $\ddot{\sigma}(0) = 0$, $\ddot{\sigma}(0) = -400$. The change of the signs in $\ddot{\sigma}(0)$ and $\dot{\sigma}(0)$ is equivalent to the inverse of time and leads to "hyperinflation".